The meaning of work: reflections on Watt, Marx, Turing and a Bee

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Abstract

The paper examines the concept of work in Watt and Marx. It asks what is it about the nature of reality that makes work possible. It approaches this via a digression on Marx's fable of the Architect and the Bee and goes on to answer it in terms of the intellectual history of thermodynamics.

Problematising labour

Prior to the eighteenth century, muscles—whether of humans, horses or oxen—remained the fundamental energy source for production. Not coincidentally, the concepts of work, power, energy and labour did not exist in anything like their modern form. People were, of course, familiar with machinery prior to the modern age. The Archimedean machines and their derivatives—levers, inclined planes, screws, wheels, pulleys—had been around for millennia to amplify or concentrate muscular effort. Water-power had been in use since at least the first century A.D.,[1] initially as a means of grinding grain; during the middle ages it was applied to a wide variety of industrial processes. But water-power, and its sister wind-power, were still special-purpose technologies, not universal energy sources. Limited by location and specialized use they did not problematize effort as such.

A note on terminology is in order here. The (admittedly not very elegant) verb 'to problematize' derives from the work of the philosopher Louis Althusser. Althusser coined the term *problématique* (problematic)(Althusser and Balibar 1970) to refer to the field of problems or questions that define an area of scientific enquiry. The term is fairly closely related to Thomas Kuhn's idea of a scientific 'paradigm'(Kuhn 1970). So, to problematize a domain is to transform it into a scientific problem-area, to construct new concepts which permit the posing of precise

[1] See (**?**), (**?**) p. 38).

scientific questions. In the pre-modern era engineers and sea captains would know from experience how many men or horses must be employed, using pulleys and windlasses, to raise a mast or obelisk. Millers knew that the grinding capacity of water mills varied with the available flow in the mill lade. But there was no systematic equation or measure to relate muscular work to water's work, no scientific problematic of effort. That had to wait for James Watt, after whom we name our modern measure of the ability to work.

Watt, the best-known pioneer of steam, did not actually invent the steam engine, but he improved its efficiency. As Mathematical Instrument Maker to the University of Glasgow he was called in to repair a model steam engine used by the department of Natural Philosophy (we would now call it Physics). The machine was a small scale version of the Newcomen engine that was already in widespread use for pumping in mines.

The Newcomen engine was an 'atmospheric engine'. It had a single cylinder, the top half of which was open to the atmosphere (Figure 1). The lower half of the cylinder was connected via two valves to a boiler and a water reservoir. The piston was connected to a rocking beam the other end of which supported the heavy plunger of a mine pump. The resting condition of the engine was with the piston pulled up by the counter-weight of the pump plunger.

To operate the machine, the boiler valve was opened first, filling the cylinder with steam. This valve was then closed and the water-reservoir valve opened, spraying cold water into the piston. This condensed the steam, resulting in a partial vacuum. Atmospheric pressure on the upper surface of the piston then drove it down, providing the power-stroke. The two phase cycle could then be repeated to obtain regular pumping.

Watt observed that the model engine could only carry out a few strokes before the boiler ran out of steam and it had to rest to 'catch its breath'. He ascertained that this was caused by the incoming steam immediately condensing on the walls of the cylinder, still cool from the previous water spray. His solution was to provide a separate condenser, permanently water cooled, and intermittently connected to the cylinder by a valve mechanism. The cylinder, meanwhile, was provided with a steam-filled outer jacket to keep its inner lining above condensation temperature (Figure 2). His 1769 patent was for "A New Method of Lessening the Consumption of Steam and Fuel in Fire Engines".

Watt's later business success was based directly on this gain in thermal efficiency. His engines were not sold outright to users, but were leased. The rental paid was equal to one-third the cost of coal saved through using a Watt engine rather than a Newcomen engine(**?**). This pricing system worked so long as the Newcomen engine provided a basis for comparison, but as Watt's engines became the predominant type, and as they came to be used to power an ever-widening

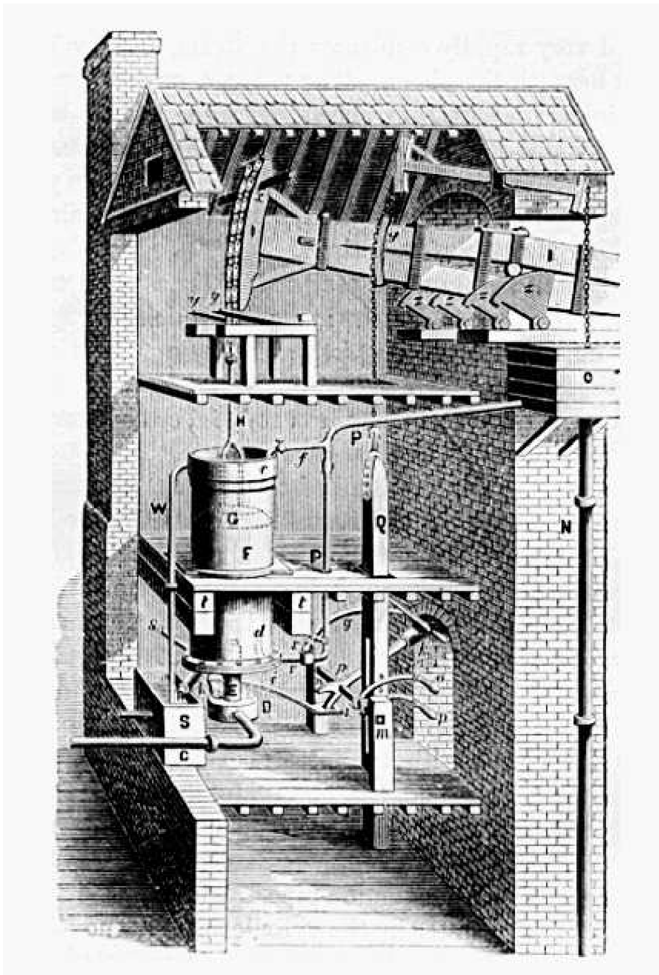


Figure 1: The Newcomen engine built by Smeaton, reproduced from (**?**).

range of machines, some system of rating the working capacity of the engines was needed. Watt needed a standardized scale by which he could rate the power, and thus the rental cost, of different engines. His standardized measure was of course the horsepower: users were charged £5 per horsepower year.

Watt's horse was not a real horse of course, but the abstraction of a horse, a standardized horse. The abstraction is multiple: at once an abstraction from particular horses, an abstraction from the difference between flesh and blood horses and iron ones, and an abstraction from the particular work done. The work done had to be defined in the most abstract terms, as the overcoming of resistance in its canonical form, namely raising weights. One horsepower is 550 ft lb/sec, the ability to raise a load of 1 ton by 15 feet in a minute.

While few real horses could sustain this kind of work, its connection to the task performed by Watt's original engines is clear. The steam engine was a direct replacement for horse-operated pumps in the raising of water from mines. But with the development of mechanisms like Watt's sun and planet gear, which converted linear to rotary motion, steam engines became a general purpose power source. They could replace water wheels in mills, drive factory machines by systems of axles and pulleys, pull loads on tracks. Engine capacity measured in horsepower abstracted from the concrete work that was being performed, transforming it all to **work** in general. Horsepower was the capacity to perform a given amount of work each second. By defining power as work done per second, work in general was itself implicitly defined. All work was equated to lifting. Work in general was defined as the product of resistance overcome, measured in pounds of force, by the distance through which it was overcome.

Mechanical power seemed to hold the prospect of abolishing human drudgery and labour. As Matthew Boulton proudly announced to George II: "Your Majesty, I have at my disposal what the whole world demands; something which will uplift civilization more than ever by relieving man of undignified drudgery. I have steam power."[2] To a world in which human muscle was a prime mover, this equation of work in the engineering sense with human labour was exact. Work on ships, in mines, at the harvest, was work in the most basic physical sense. Men toiled at windlasses to raise anchors, teams pulled on ropes to set sails and hauled loads on their backs to unload cargo. Children dragged coal in carts from drift mines,

[2] Compare Antipater of Thessalonika's eulogy on the introduction of the water mill:

> Stop grinding, ye women who toil at the mill
> Sleep on, though the crowing cocks announce the break of day
> Demeter has commanded the water nymphs
> to do the work of your hands
> Jumping one wheel they turn the axle
> Which drives the gears and the heavy millstones

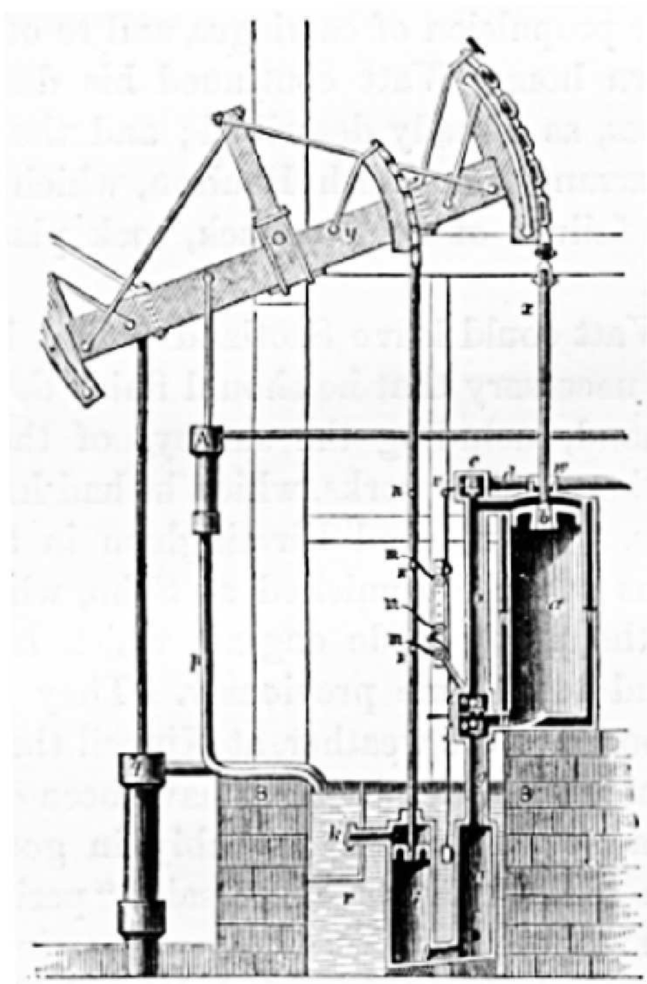


Figure 2: Watt’s steam engine with separate condenser from (**?**)

women carried it up shafts in baskets on their backs. The 'navigators' who built canals did it with no mechanical aid more sophisticated than the wheelbarrow (a combination of lever and wheel, two Archimedean devices).

As horsepower per head of population multiplied, so too did industrial productivity. The power of steam was harnessed, first to raise weights, then to rotate machinery, then to power water-craft, next to trains—and eventually, through the mediation of the electricity grid, to tasks in every shop and home—while human work shrank as a proportion of the total work performed. More and more work was done by artificial energy, yet the need for people to work remained. A steam locomotive might draw a hundred-ton train, but it needed a driver to control it. Human work became increasingly a matter of supervision, control and feeding of machines. Thus the identification of work with the overcoming of physical resistance in the abstract, and of human labour-power with power in Watt's sense, contained both truth and falsehood. Its truth is shown by the manifest gains flowing from the augmentation of human energy. Its falsity is exposed by the residuum of human activity that expresses itself in the control, minding and direction of machinery.

Indeed, the introduction of powered machinery had the effect of lengthening the working day while making work more intense and remorseless. The cost of powered machinery was such that only men with substantial wealth could afford it. Cheap hand-powered spindles and looms could not compete with steam-powered ones. Domestic spinners and hand-loom weavers had to give up their independence and work for the owners of the new steam powered 'mules' and looms. Steam power brought no increase in leisure for weavers or spinners. The drive to recoup the capital cost of the new machinery brought instead longer working hours and shift-work, to a rhythm dictated by the tireless engine. The fact that the machinery was not owned by those who worked it, meant that it enslaved rather than liberated.

A particular pattern of ownership was the social cause of machine-enforced wage slavery, but that is only half the story. We may ask why the new machine economy needed human labour at all. Why did 'self acting'—or as we would put it now, 'automatic'—machines not displace human labour altogether? A century ago, millions of horses toiled in harness to draw our loads. Where are they now? A remnant of their former race survives as toys of the rich; the rest went early to the knackers. Why has a similar fate not befallen human workers? Why has the race of workers not been killed off, to leave a leisured rich attended by their machines?

Watt's horsepower killed the horse, but the worker survived. There must be some real difference between work as defined by Watt, and work in the sense of human labour.

Marx: The Architect and the Bee

Karl Marx proposed an argument which seems at first sight to get to the essence of what distinguishes human labour from the work of an animal or a machine, namely purpose.

> An immeasurable interval of time separates the state of things in which a man brings his labour-power to the market for sale as a commodity, from that state at which human labour was still in its first instinctive stage. We pre-suppose labour in a form which stamps it as exclusively human. A spider conducts operations that resemble those of a weaver, and a bee puts to shame many an architect in the construction of her cells. But what distinguishes the worst of architects from the best of bees is this, that the architect raises his structure in the imagination before he erects it in reality. At the end of every labour process we get a result that already existed in the imagination of the labourer at its commencement. He not only effects a change of form in the material on which he works, but he also realises a purpose of his own that gives the law to his modus operandi, and to which he must subordinate his will. ((**?**) pp. 177–8)

This suggests that animals, lacking purpose, can be replaced by machines, but that humans are always required, in the end, to give purpose to the machine. We cite Marx's statement because it articulates what is probably a rather widely held view, yet it has several interesting problems. This is an issue where it is difficult to go straight for the 'right answer'. It may be profitable to beat the bushes first, to scare up (and shoot down) various prejudices that can block the road to a scientific understanding.

First, are animals really lacking in purpose? The spider may be so small, and her brain so tiny, that it seems plausible that blind instinct, rather than the conscious prospect of flies, drives her to spin. But it is doubtful that the same applies to mammals. The horse at the plough may not envisage in advance the corn he helps to produce, but then he is a slave, bent to the purpose of the ploughman. Reduced to a source of mechanical power, overcoming the dumb resistance of the soil, he is readily replaced by a John Deere. The same cannot be said of animals in the wild. Does the wolf stalking its prey not intend to eat it? It plans its approach with cunning. Who are we to say that the result—fresh caribou meat—did not "already exist in the imagination" of the wolf at its commencement? We have no basis other than anthropocentric prejudice on which to deny her imagination and foresight.

Turn to Marx's human example, an architect, and his argument looks even shakier. For do architects ever build things themselves? They may occasionally build their own homes, but in general what gives them the status of architects is that they don't get their hands dirty with anything worse than India Ink. Architects draw up plans. Builders build. (In eliding this distinction Marx showed an uncharacteristic blindness to class reality).

An office block, stadium or station has, it is true, some sort of prior existence, but as a plan on paper rather than in the mind of the builders. If by collective labour civilized humans can put up structures more complex than bees, it is because they can read, write and draw. A plan—whether on paper or, as in earlier epochs, scribed on stone—coordinates the individual efforts of many humans into a collective effort.

For building work then, Marx is partially right, the structure is raised *on paper* before it is raised in stone. But he is wrong in saying that it is built in the imagination first, and in implying that the structure is put up by the architect. What is really unique to humans here is, first, the social division of labour between the labour of conception by the architects and the work of execution by the builders, and second, the existence of *materialized plans*: configurations of matter that can control and direct the labour of groups of humans.

While insect societies may have a division of labour between 'castes', for example between worker and soldier termites, they do not have a comparable division between conception and execution, between issuers and followers of orders. Nor do insects have technologies of record and writing. They can communicate with each other. Dancing bees describe to others the whereabouts of flowers. Walking ants leave scent trails for their companions. These messages, like human speech, coordinate labour. Like our tales, they vanish in the telling. But, not restricted to telling tales, we can can make records that persist, communicated over space and time.

Our tales are richer too. The set of messages that can be expressed in our languages is exponentially greater than in the language of bees. Each works by the sequential combination of symbols—words for us, wiggles for bees—but we have many more symbols and can understand much longer sequences. The number of distinct messages that can be communicated by a language is proportional to v^m where v is the number of distinct symbols that can be recognized in the language and m is the maximum message length. If bees have a repertoire of six types of wiggles and can understand 'sentences' of three wiggles in succession then they can send $6^3 = 216$ different messages. A human language with a vocabulary of 3000 words and a maximum sentence length of 20 words could convey about $3.486784401 \times 10^{69} = 348,678,440,100,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000$ distinct sentences. Of course, not all of these would be grammatically correct, and a rather small proportion of those would make any sense, but the number of messages is still astronomically greater than what insects can manage. And we can keep piling on the sentences until the listener loses track.

All this leaves open another interpretation of what Marx had to say. True enough, architects may not build theatres themselves, any more than Hadrian built his wall[3] or Diocletian his baths. But Hadrian caused the wall to be built and Diocletian's architect caused the baths to be built to a specific design. (This use of the word 'built' is of course common in class societies, where real builders get no credit for their creations. Their labour contributes instead to the fame of a ruler or architect.) If the architect creates only a paper version of a theatre, can we say, at any rate, that he creates this drawing in his mind before setting it down on paper? This interpretation of Marx's story of the architect and the bee seems to make sense, but it's not clear that it's a true description of what an architect actually does.

Emergent buildings

Some individuals, autistic infant prodigies or 'idiot savants', do seem to have the ability to hold in their minds almost photographically detailed images of buildings they have seen. Working from memory they are able to draw buildings in astonishing and accurate detail. But it is questionable whether professional architects work this way. Some may, but for others the process of developing a design is intimately tied up with actually drawing it. They start with the broad outlines of a design in their minds. As this is transferred to paper, they get the contexts within which the mind can work to elaborate and fill in details. The details were not in the mind prior to starting work, they emerge through the interaction of mind, pen and paper. Pencils and paper don't just record ideas that exist fully formed, they are part of a production process that generates ideas in the first place.

At any one time our consciousness can focus on only a limited number of items. On the basis of what it is currently conscious of, its context, it can produce responses related to this context. In reverie the context is internal to the brain and the responses are new ideas related to this context. In an activity like drawing a plan or engineering diagram, the context has two parts

(1) an internal state of mind; and

(2) that part of the diagram upon which visual attention is fixated,

and the response is both internal—a new state of mind—and external—a movement of the pencil on the paper.[4] Where in reverie the response, the new idea, slipped all too easily from grasp, paper remembers.Architecture exchanges for the

[3]It was of course the rank and fi le legionnaires who built the wall; see (Davies 1989).

[4]The reader may notice that this argument is a thinly disguised version of Alan Turing's famous argument (**?**).

fallibility and limited compass of memory the durability of an effectively infinite supply of A0. One might say that complex architecture rests on paper foundations.

If the idea of the architect as creating buildings spontaneously out of the imagination is dismissed as an almost religious myth, redolent of the Masonic characterization of the deity as the *Great Architect*, what then remains of the antithesis between architect and bee? Well, how do the bees shape their hive? We can be sure there are no drawings of hexagons, made by the 'queen',[5] and executed by her worker daughters. We are talking here of *apis mellifera* not the solitary bumble bee. The labour of the honey bees is collective, like that of workers on a building site, yet although they have no written plans to work from they create a geometrically precise, optimal and elegant structure.

Apian efficiency

Consider the problem to which the honeycomb is the answer: to come up with a structure that is interchangeably capable of storing honey or sheltering bee larvae, is waterproof, is structurally stiff, provides a platform to walk on and which uses the minimum material. Given this design brief it is unlikely that a human engineer could come up with a better structure.

The structure has to be organized as a series of planes to provide access. Within the planes, the combs, the space has to be divided into approximately bee-sized cubicles. These could be triangular, square, or hexagonal (the only three regular tessellations of the plane). Our architects have a predilection for the rectilinear, but the hexagonal form is superior.

Figure 3: Tessellation of the plane using hexagons

[5]The breeding female is no more an architect or Caesar than the Pope is the genetic father of his followers. Monarchy and patriarchy project dominance relations onto genetic relations and vice versa. Apian Mother becomes queen, the Vatican monarch, Holy Father.

A tessellation of unit squares has a wall length of 2 per unit area, since a single unit square has four sides of unit length, each shared 50 percent with its neighbours. A tessellation of hexagons of unit area has a wall length of $\frac{2}{\sqrt{3}}$ per unit area, a reduction by a factor of $\sqrt{3}$ (see Digression 0.1). The honeycomb structure used by bees is thus more efficient in its use of wax than a rectilinear arrangement would be.

The fact that hexagonal lattices minimize boundary lengths per unit area means that they can arise spontaneously, for example in columnar basalts. Here the tension induced in rocks as they cool encourages cracking, preferentially giving rise to six sided columns. We might suspect that the beehive too, gained its structure from a process of spontaneous pattern formation analogous to columnar basalts or packed arrays of soap bubbles. But this doesn't tally with the way the cells are built up, or with the uniformity of their dimensions. In a partially constructed honeycomb the cells are of a constant diameter; those in the middle of the comb are all of uniform height while towards the edge the depth of the cells falls. The bees build the cells up from the base, laying wax down on the upper margins of the cell walls, just as bricks are added to the upper margin of a wall by a bricklayer. The construction process takes advantage of the inherent stability of a hexagonal lattice, allowing the growing cells to form their own scaffolding. But the process also demands that the bees can deposit wax accurately on the growing cell walls, and that they stop building when the cells have reached the right height. That is, it depends on purposeful activity on the part of the bees.

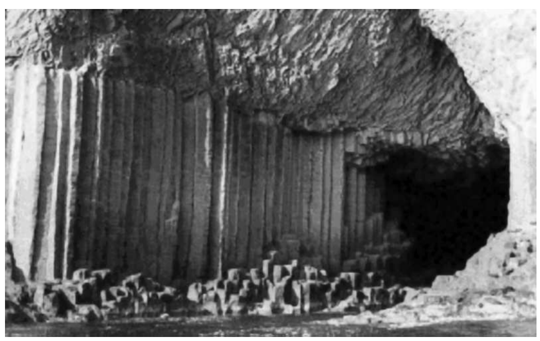
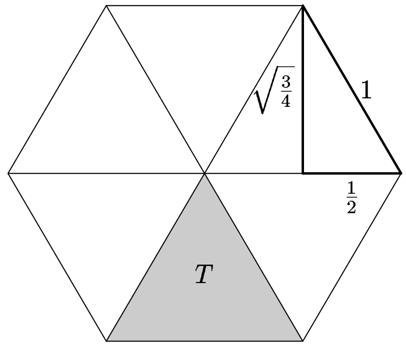


Figure 4: Nature is the architect of the hexagonal columns of Fingal's cave (Photo by Andrew Kerr)

A similar process takes place in the human construction of geodesic domes, hexagonal lattices curved through a third dimension. These have an inherent sta-

Digression 0.1 Apian efficiency



(1) A hexagon of unit side is made up of 6 identical equilateral triangles, thus its area is $6T$ where T is the area of an equilateral triangle of unit side.

(2) The area of an equilateral triangle of unit side is $\frac{1}{2}bh$ where b the base $= 1$ and h the height $= \sqrt{\frac{3}{4}}$. So $T = \frac{1}{2}\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{4}$.

(3) The area of one hexagon is then

$$6\frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$$

(4) The hexagon's six sides are each shared 50% with a neighbour.

(5) Wall per unit area for a hexagonal tessellation is then $3/\frac{3\sqrt{3}}{2} = 2/\sqrt{3}$ which is better than the wall to area ratio for squares.

The Honeycomb Conjecture has been debated since at least 36BC when it was mentioned by Varro in his book on agriculture. It has been remarkably difficult to prove. Here we have considered only a comparison between hexagonal tesselations and square ones. There remains the possiblity that some layout using curved walls might be still more efficient. A full proof of the conjecture was not produced until 2001 (Hales 2001).

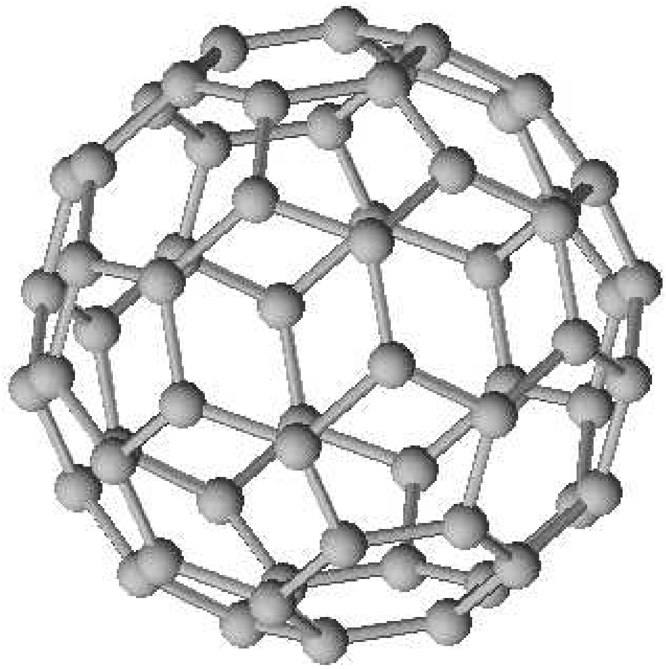


Figure 5: C_{60} a spontaneously formed dome structure

bility that becomes more and more evident as you add struts to them. You build them up in a ring starting at ground level. The structure initially has a fair bit of play in it, but the closer the structure comes to a sphere the more rigid it is. Human dome builders, like bees, exploit the inherent structural properties of hexagonal lattices, but they still need to cut struts to the right length and put them in the correct place. The bees likewise must select the right height for their cell walls and place wax appropriately.

Spontaneous self-assembly of hexagonal structures similar to geodesic domes does occur in nature. The Fullerenes are a family of carbon molecules named after Buckminster Fuller, the inventor of the geodesic dome. The first of these to be discovered, C_{60}, has the form of a perfect icosahedron (see Figure 5). Condensed out of the hellish heat of a carbon arc, it depends on thermal vibrations to curve the familiar planar hexagonal lattice of graphite onto itself to form a three dimensional structure. No architect or bee is required. Atomic properties of carbon select the strut length. Thermal motion searches the space of possible configurations; a small fraction of the molecules settle into the local energy minima represented by C_{60} and its sisters.

If the bees can't rely upon spontaneous self-assembly to build their hives, must they have a plan in mind before they start? Since they can't draw, the mind would have to be where they held any plans. While we can't rule this out, it seems unlikely. The requirement is that they can execute a program of work. A bee arriving on the construction site with a load of wax must, in the darkness, find an appropriate place to put it, for which they need a set of rules:

> If the cell is high enough to crawl into, put no more wax on it,
> otherwise if the cell has well formed walls add to their height,
> otherwise if it is a cell base smaller than your own body diameter, expand it,
> otherwise start building the wall up from the base. . .

No internal representation of a completed comb need be present in the bee's mind. The same rules, simultaneously present in each of a hive full of identical cloned sisters, along with the structural properties of beeswax, produce the comb as an emergent complex structure. The key here is the interaction between behavioral rules and an immediate environment that is changed as the result of the behaviour. The environment, the moulded wax, records the results of past behaviour and conditions future behaviour. But for rules to be converted into behaviours by the bees, the bees must have internal 'states of mind', and be able to change their state of mind in response to what their senses are telling them. A bee that is busy laying down wax is in a different state of mind from one foraging for pollen and their behavioral repertoire differs as a result.

As we have argued above, what an architect does is not so different. Architects produce drawings, not buildings or hives, but producing a drawing is an interactive process in which the architect's internal state of mind, his knowledge of the rules and stylistic conventions of the epoch, produces behaviour that modifies the immediate environment—the paper. The change to the paper creates a new environment, modifying his state of mind and calling into action other learned rules and skills. The drawing is an emergent property of the process, not something that pre-existed as a complete internal representation before the architect put pencil to paper.

The Demonic challenge

Purposeful labour depends upon the ability to form and follow goals. A goal is a representation of a state of affairs that does not exist plus a motivation to achieve it. Although bees do not have the goal processing capabilities of the human mind, they nonetheless follow simple goals. Goal processing, from simple, reactive programs hard-wired in the neural circuitry of insects, to the much more adaptive and sophisticated rational planning capabilities of humans, is the mechanism that distinguishes the constructive activity of humans and bees from the blind efforts of Watt's engines. An engine transforms energy in one form to another, but it does not act to achieve states of affairs, unlike bees that build or humans that labour.

There is a hidden connection between purposeful labour and work in the engineering sense. Any purposeful activity overcomes physical resistance and involves *work*, measured in watts, for which we must be fueled by calories in our food; the hidden connection comes from the realization that, at least in principle, purposeful labour could itself be a source of fuel.

Recall that Watt's key invention was the separate condenser for steam engines, which saved fuel by preventing wasteful condensation of steam within the cylinder of the engine. In the years after Watt's invention, it came to be realized that the thermal efficiency of steam engines could be improved by maximizing the pressure drop between the boiler and the condenser. A series of inventions followed to take advantage of this principle: Trevithick's high pressure engine, the double and then the triple expansion engine. These had the effect of increasing the amount of effective work that could be extracted from a given amount of heat. But successive gains in efficiency proved harder to come by. The amount of work obtained per calorie of heat could be increased, but not without limit.

It was understood that work could be converted into heat, for instance through friction, and heat could be converted back into work, for instance by a steam engine. But if you convert work into heat, and heat back into work, you always end up with less work than you put in. In converting work into heat, the number of calories of heat obtained per kilowatt hour of work is constant—conversion of work into heat can be done with 100 percent efficiency. The reverse is not true. Heat can never be fully converted into useful work.[6] The practical imperative of improving steam engines gave rise to the theoretical study of the laws governing heat, the laws of thermodynamics.

One of the first formulations of the second law of thermodynamics was that heat will never spontaneously flow from somewhere cold to somewhere hot.[7] This implied that, for instance, there was no chance of transferring the heat wasted in the condenser of a steam engine back to the boiler where it would boil more water. Thermodynamics ruled out perpetual motion machines.

But James Clerk Maxwell, one of the early researchers in thermodynamics, came up with an interesting paradox.

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[6] Carnot was able to show that the effi ciency of heat engines depended on the temperature difference between heat source, for example the boiler, and the heat sink, for example a steam engine's condenser.

[7] This formulation was due to Clausius in 1850; see ((**?**) pp. 8–9).

> One of the best established facts of thermodynamics is that it is impossible in a system enclosed in an envelope which permits neither change of volume nor passage of heat, and in which temperature and pressure are everywhere the same, to produce any inequality of temperature or of pressure without the expenditure of work. This is the second law of thermodynamics, and it is undoubtedly true as long as we can deal with bodies only in mass, and have no power of perceiving or handling the separate molecules of which they are made up. But if we can conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being would be able to do that which is presently impossible to us. For we have seen that the molecules in a vessel full of air at a uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower ones to pass from B to A. He will thus, without the expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics. (**?**)(pp. 328–329)

The configuration of the thought experiment is shown in Figure 6. As the experiment runs the gas on one side heats up while that on the other side cools down. The end result is a preponderance of slow molecules in cavity A, fast ones in cavity B. Since heat is nothing more than molecular motion, this means that A has cooled down while B has warmed up. No net heat has been added, it has just re-distributed itself into a form that becomes useful to us. Since B is hotter than A, the temperature differential can be used to power a machine. We can connect B to a boiler and A to a condenser and obtain mechanical effort. An exercise of purposeful labour by the demon outwits the laws of thermodynamics. (Norbert Wiener coined the term ‘Maxwell demon’ for the tiny ‘being’ envisaged in the thought experiment.) It seems that the second law of thermodynamics expresses the coarseness of our senses rather than the intractability of nature.

Entropy

One perspective on the devilment worked by Maxwell’s demon is that it has *reduced the entropy* of a closed system. The idea of entropy was introduced by Clausius in 1865 (Harrison 1970) with the equation

$$\Delta S = \Delta Q/T \tag{1}$$

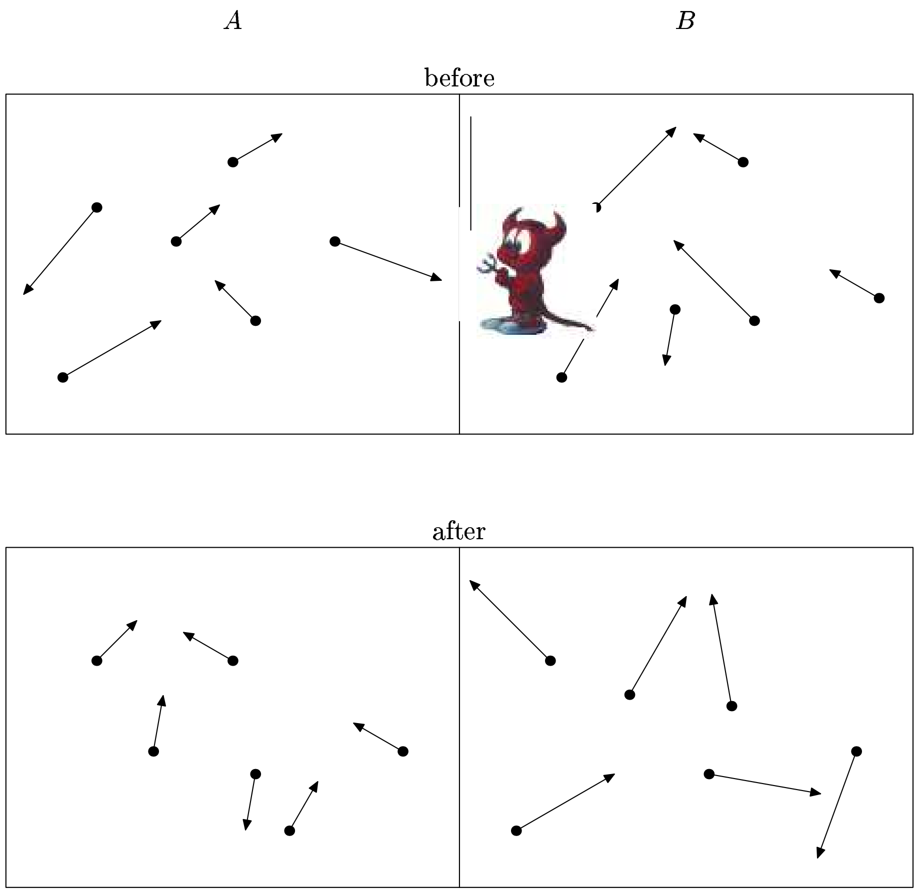

Figure 6: Gas initially in equilibrium. Demon opens door only for fast molecules to go from A to B, or slow ones from B to A. Result Slow molecules in A, fast in B. Thus B hotter than A, and can be used to power a machine.

where ΔS is the change in entropy of a system consequent upon the addition of a quantity of heat ΔQ at absolute temperature T.[8] According to Clausius's equation adding heat to a system always increases its entropy (and subtracting heat always lowers entropy) but the magnitude of the change in entropy is inversely related to the initial temperature of the system. Thus if a certain amount of heat is transferred from a hotter to a cooler region the increase in entropy in the cooler region will be greater than the reduction in entropy in the hotter, and overall entropy rises. Conversely, if heat is transferred from a colder to a hotter region entropy falls. Clausius's concept of entropy as an abstract quantity allowed him to give the second law of thermodynamics its canonical form: the entropy of any closed system tends to increase over time.

Using (1) we can readily see that Maxwell's demon violates the second law of thermodynamics. Suppose the demon has been hard at work for some time, so that B is hotter than A, specifically B is at 300° Kelvin and A is at 280° Kelvin. He then transfers ΔQ =1 joule of heat from A to B. In doing so he reduces the entropy of A by $\frac{1}{280}$ joules per degree and increases the entropy of B by $\frac{1}{300}$ joules per degree giving rise to $\Delta S = \frac{1}{300} - \frac{1}{280} = -\frac{1}{4200}$, a net reduction in entropy, contrary to the second law.

Clausius's formulation of entropy did not depend in any way upon the atomic theory of matter. Maxwell's proposed counter-example to the second law was explicitly based on atomism. With Boltzmann, entropy is placed on an explicitly atomistic foundation, in terms of an integral over molecular *phase space*.

$$S = -k \int f(v) \log f(v) dv \tag{2}$$

where v denotes volume in six-dimensional phase space, $f(v)$ is the function that counts the number of molecules present in that volume, and k is Boltzmann's constant.

The concept of phase space is a generalization of our normal concept of three-dimensional space to incorporate the notion of motion as well as position. In a three-dimensional coordinate system the position of each molecule can be described by three numbers, measurements along three axes at right angles to one another. We usually label these numbers x, y, z to denote measurements in the horizontal, vertical and depth directions. However each molecule is simultaneously in motion. Its motion can likewise be broken into components of horizontal, vertical and depth-wise motion which we can write as m_x, m_y, m_z, representing motion to

[8] At this stage the concept of entropy remains firmly linked to the sort of practical considerations, namely steam engine design, that gave rise to thermodynamics. Later, as we shall see, it becomes generalized.

the left, up and back respectively. This means that a set of six coordinates can fully describe both the position and motion of a particle.

In Boltzmann's formula, the letter v denotes a range of possible values of these co-ordinates. For example, a volume 1 mm cubed on the spatial axes and 1 mm per second on the motion axes. The function $f(v)$ would then specify how many molecules there were in that cubic millimeter with a range of velocities within 1 mm per second in each direction. Boltzmann's formula relates the entropy of a gas, for instance steam in a piston, to the evenness of its distribution in this six dimensional space: the less even the distribution the lower the entropy. This point is illustrated in simplified manner in Table 1. Suppose we have just two cells in phase space, and eight atoms that can be in one cell or the other. The table shows how the entropy depends on the location of the atoms, lowest when all 8 are in one cell, and highest when they are evenly divided between the cells. (Note that the minus sign in Boltzmann's formula is needed to make entropy increase with the evenness of the distribution, consistent with Clausius's earlier formulation.)

Contents of cells 1, 2	$f(1)\log f(1) + f(2)\log f(2)$	Entropy, S
8, 0	8(2.079) + 0 = 16.636	$-16.636k$
7, 1	7(1.946) + 1(0) = 13.621	$-13.621k$
6, 2	6(1.792) + 2(0.693) = 12.137	$-12.137k$
5, 3	5(1.609) + 3(1.099) = 11.343	$-11.343k$
4, 4	4(1.386) + 4(1.386) = 11.090	$-11.090k$

Table 1: Boltzmann's entropy: Illustration

Boltzmann also showed that it is possible to reformulate the idea of entropy using the concept of the 'thermodynamic weight' of a state:

$$S = k \log W \tag{3}$$

The thermodynamic weight W is the number of physically distinct microscopic states of the system consistent with a given 'macro' state, described by temperature, pressure and volume. This concept is the key to understanding the second law. Recall that the entropy of closed systems tends to increase, that is they move into macro-states of progressively higher thermodynamic weight until they reach equilibrium. States with higher weight are *more probable.* So the second law of thermodynamics basically says that systems evolve into their most probable state.

A simple analogy may be helpful here. Suppose a 'fair' coin is flipped ten times. What is the most likely ratio of heads to tails in the sequence of flips? The obvious answer, 5/5, is correct. Now, what is the most likely specific sequence of heads and tails? Trick question! There are $2^{10} = 1024$ such sequences and they are all equally likely. The sequence featuring 10 heads has probability $\frac{1}{1024}$; so does the sequence with 5 heads followed by 5 tails; so does the sequence of strictly alternating heads and tails, and so on. The reason why a 5/5 ratio of heads to tails is most likely is that there are more specific sequences corresponding to this ratio that there are sequences corresponding to 10/0, or 7/3, or any other ratio.

It's easy to see there is only one sequence corresponding to all heads, and one corresponding to all tails. To count the sequences that give a 5/5 ratio, imagine placing the 5 heads into 10 slots. Head number 1 can go into any of the ten slots; head number 2 can go into any of the remaining 9 slots, and so on, giving $10 \times 9 \times 8 \times 7 \times 6$ possibilities. But this is an over-statement, because we have treated each head as if it were distinct and identifiable. To get the right answer we have to divide by the number of ways 5 items can be assigned to 5 slots, namely $5 \times 4 \times 3 \times 2 \times 1$. This gives 252 possibilities. Thus the 'macro' result, equal numbers of heads and tails, corresponds to 252 out of the 1024 equally likely specific sequences, and has probability $\frac{252}{1024}$. By the same reasoning we can figure that a 6/4 ratio corresponds to 210 possible sequences, a lower 'weight' than the 5/5 ratio.

The number of possible states of a real gas in six-dimensional phase space is hard to visualize, so to explicate the matter further we'll examine a simpler system, namely a two-dimensional *lattice gas* (Frisch, Hasslacher, and Pomeau 1986). The 'molecules' in such a stylized gas move with constant speed, one step along the lattice per unit time (see Figure 7). Where the lines of the lattice meet, molecules can collide according to the rules of Newtonian dynamics, so that matter, energy and momentum are conserved in each collision. The different ways in which collisions occur can be summarized by two simple rules:

(1) If a molecule arrives at an intersection and no molecule is arriving on the diagonally opposite path, then the molecule continues unimpeded.

(2) If two molecules collide head on they bounce off in opposite directions, as shown in Figure 8.

Lattice gases are a drastic simplification of real gases, but they are useful tools in analysing real situations. The simple rules governing the behaviour of lattice gases make them ideal models for simulation in computer software or special purpose hardware (**?**).

Since the velocity of the molecules in a lattice gas is fixed, the temperature of the gas can't change (this would involve a rise or fall in the molecules' speed). So Maxwell's original example of a being with precise senses, able to sort molecules by speed, is inappropriate. But we can invent another demon to guard the trapdoor. Instead of letting only fast molecules through from A to B, this being will

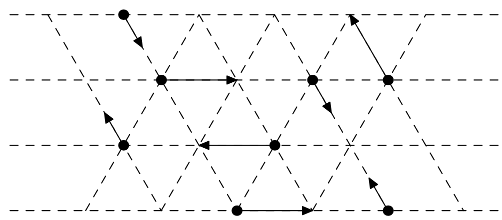

Figure 7: The molecules in a lattice gas move along the lines of a triangular grid with fixed velocities

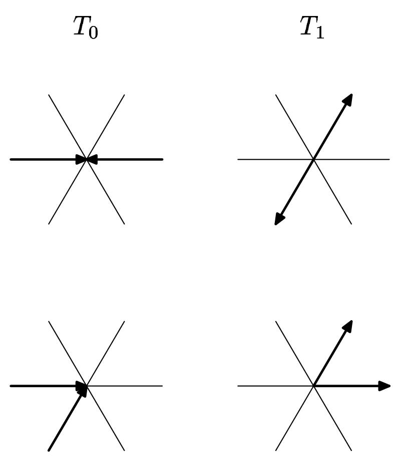

Figure 8: Collisions in a lattice gas: ‘Molecules’ colliding head on bounce off at 60° angles (above). In other cases the collision is indistinguishable from a miss (below). In all cases Newtonian momentum and energy are conserved.

keep the door open unless a molecule approaches it from side B. Thus molecules approaching from side A are able to pass into B, but those in B are trapped. The net effect is to raise the pressure on side B relative to A while leaving temperature unchanged.

A lattice gas has only a finite number of lattice links on which molecules can be found, and since the molecules move with a constant velocity, Boltzmann's formula (3) simplifies to:

$$S = -kn \sum_i p_i \log p_i \tag{4}$$

where p_i is the probability of the node being in state i and n is the number of nodes. The weighted summation over the possible states has the effect of giving us the mean value of $\log p$. Suppose we have a very small pair of chambers, A and B, each of which initially has n nodes, and each containing $3n$ randomly distributed molecules. Then each of the six incoming paths to a node will have a 50 percent chance of having a molecule on it. We have $6n$ incoming paths to our nodes, and each of these has two equally likely states: a particle is or is not arriving at each instant. Each incoming path contributes $k \log 2 = 0.693k$. The total entropy of the chamber is then six time this or:

$$\text{Entropy of A in equilibrium} = 4.158kn.$$

Now suppose that our demon has been operating for some time, letting n particles pass from A to B, so that A now contains $2n$ particles and B contains $4n$ particles. In A, the probability of a molecule coming down any one of the paths is now only $\frac{1}{3}$. We can calculate the current entropy contribution of each incoming path as follows:

Number of particles	probability, p_i	$\log p_i$	entropy, $-kp_i \log p_i$
0	$\frac{2}{3}$	-0.405	$0.27k$
1	$\frac{1}{3}$	-1.098	$0.366k$
total			$0.636k$

The entropy of A after n particles have been transferred by the demon is $3.816kn$ which is less than before he got to work. By symmetry of complementary probabilities the entropy of chamber B will be the same,[9] thus the whole closed system has undergone a reduction in entropy.

This establishes that when an initially dispersed population of particles—the gas molecules in our case—is concentrated, entropy falls.[10] This is because there

[9] This will not generally be the case; we have chosen the particle densities so as to ensure this.

[10] This is true on the assumption that the potential, gravitational or electrostatic, of the particles is unchanged by the process of concentration as in our example.

are a greater number of possible microstates compatible with dispersion than with concentration, and entropy is just the log of the number of microstates.

Consider in this light the work of the bees building their hive. There are two aspects to the work:

(1) The bees first have to gather wax and nectar from flowers dispersed over a wide area and bring it to the hive.
(2) They must then form the wax into cells and place the concentrated nectar in these as honey.

Both processes are entropy-reducing with respect to the wax and the sugar. The number of possible configurations that can be taken on by wax within the few litres volume of a hive is enormously less than the number of possible configurations of the same wax, dispersed among plants growing over tens of thousands of square meters of ground. Similarly the chance that the wax, if randomly thrown together within the hive, should assume the beautifully regular structure of a comb, is vanishingly small. That the wax should be in the hive in the first place, is, in the absence of bees, highly improbable; that it should be in the form of regular hexagons even more so.

The second law of thermodynamics specifies that the total entropy in a closed system tends to increase, but the bees and their wax are not a closed system. The bees consume chemical energy in food to move the wax. If we include the entropy increase due to food consumed, the second law is preserved.

Men and horses

Let us return to the question we asked in section : Why did the introduction of the steam engine, which made redundant the equine workers of the pre-industrial age, not also replace the human workers? We can make a rough analogy between the work done by horses in past human economies and the work done by the bees in transporting wax and nectar from flower to hive. This is in the main sheer effort, work in Watt's sense. Horses bringing bricks to a building site or bees transporting wax are doing similar tasks. What remains, the construction of the hive after the work of transportation is done or the building of the house once the bricks are delivered, is something no horse can do. Construction involves a complex program of actions deploying grasping organs, hands, mandibles, beaks etc., in which the sequence of operations is conditioned by the development of the product being made. Human construction differs from that of a bee or a bird in:

(1) the way in which the program of action comes into being;
(2) the way in which it is transmitted between individuals of the species; and

(3) the form in which it is materialized.

In the social insects the programs of action largely come into being through the evolutionary process of natural selection. They are transmitted between parents and their offspring genetically encoded in DNA, and they are materialized in the form of relatively fixed interactions between components of the nervous system and general physiology. In humans the programs of action are themselves products that can have a representation external to the organism, in speech or some form of notation. Speech and notation act both as a means of transmission between individuals, and as a possible form of materialization of work programs while the work is being carried out—as for example, when one cooks from a recipe or follows a knitting pattern. *The ability to make and distribute new work programs distinguishes human labour from that of bees and is the key to cultural evolution.*

But even the work of transport requires a program of action, requires guidance if it is to reduce entropy. Transport is not diffusion. It moves concentrated masses of material between particular locations, it does not spread them about willy nilly. Without guidance there is no entropy reduction. A horse, blessed with eyes and a brain as well as big muscles, will partially steer itself, or at least will do better than a bicycle or car in this respect. But teams still needed teamsters, if only to read signposts.

The steam railway locomotive revolutionized land transport in the nineteenth century, quickly replacing horse traction for long overland journeys. Guidance by steel track made steam power the great concentrator, bringing grain across prairies to the metropolis. Railway networks are action programs frozen in steel, their degrees of freedom discrete and finite, encoded in points. Point settings, signaled by telegraph, coordinate the orderly movement of millions of tons according to precise published timetables.

Human work did not all lend itself so readily to mechanization.

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