

# Did Marx have a labour theory of value?

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## Abstract

It has long been accepted that Marx was a follower of the labour theory of value. This position has recently been challenged by Harvey. This paper shows that the essential basis of Marx's value theory remained labour and that it did not differ substantially from that of Ricardo. It also references data showing the empirical validity of the theory, and presents example data from the UK 1998 input output table showing how closely monetary output shadows labour content.

## 1 Introduction

Did Marx have a labour theory of value?

It seems absurd that one has to answer this question.

It was so thoroughly answered by previous generations of economists in the affirmative Meek ((1956)), that it would seem unnecessary even to examine the issue. But David Harvey has recently posted a short article Harvey ((2018)) claiming that Marx was an opponent of the labour theory

It is widely believed that Marx adapted the labour theory of value from Ricardo as a founding concept for his studies of capital accumulation. Since the labour theory of value has been generally discredited, it is then often

authoritatively stated that Marx's theories are worthless. But nowhere, in fact, did Marx declare his allegiance to the labour theory of value. That theory belonged to Ricardo, who recognized that it was deeply problematic even as he insisted that the question of value was critical to the study of political economy. On the few occasions where Marx comments directly on this matter,<sup>1</sup> he refers to "value theory" and not to the labour theory of value. So what, then, was Marx's distinctive value theory and how does it differ from the labour theory of value?( Harvey ((2018)))

Later he writes:

Marx appears to have done little more than synthesize and formalize Ricardo's labour theory of value by embedding it in the totality of circulation and accumulation .... The sophistication and elegance of the argument have seduced many of Marx's followers to thinking this was the end of the story. If this was so then much of the criticism launched against Marx's theory of value would be justified. But this is not the end. It is in fact the beginning. Ricardo's hope was that the labour theory of value would provide a basis for understanding price formation. It is this hope that subsequent analysis has so ruthlessly and properly crushed.

Harvey claims that the labour theory of value is generally discredited. But in what sense?

In what way has subsequent analysis' ruthlessly and properly crushed' the labour theory of value?

It is correct to say that the theory is not viewed with favour in economics departments, but that is for political reasons - the labour theory of value came, since Gray and Marx, to be associated with socialism. Since academic economists in general did not want to be tainted with the socialist label they were at pains to distance themselves from the theory. But none of them ever adduced any empirical evidence to refute it. It was

socially discredited but not empirically refuted.

If one wants to refute a theory about the world you have to show that the theory makes incorrect empirical predictions. Eratosthenes refuted the theory that the Earth was flat and confirmed the theory that it is round by observing that when the sun was overhead in Syene it was at an angle of  $7^{\circ}12'$  to the vertical in Alexandria, implying that the earth was curved with a circumference of 25,000 miles. If the earth had been flat the angle of the sun would not have varied as you went north.

## 2 Evidence

The labour theory of value predicts that the prices of commodities will vary proportionately with their labour content. Refuting this should be easy, just show that in fact their prices do not vary with labour content in this way. Did the economists opposed to the labour theory of value do this?

Did they hell! They did not even bother to collect the data to do the tests. So for a century after Marx, the theory was 'discredited', but never empirically refuted.

As soon as economists started to collect data to test the theory, which depended on reasonably good economic statistics of whole economies, what did they find?

They found the Ricardo and Marx had been right all along. A whole bunch of studies Zachariah ((2004)); Tsoulfidis and Paitaridis ((2016)); Fröhlich ((2013)); Ochoa ((1989)); Gintis and Bowles ((1981)); Petrovic ((1987)); Shaikh ((1998)); Zachariah ((2006)); Cockshott et al. ((1993)); Cockshott and Cottrell ((1997,)) or ((Shaikh, 2016, pp. 69-70.)) since the 1980s have shown that the labour theory of value is very good at predicting prices.

The technique used in all of these studies is to use Input/Output (IO) tables to work back and find out how much direct and indirect labour goes into the output of each industry. Technically this can be done either by a Jacobi iterative solution for labour values or by matrix inversion. One then either regresses the labour content of

the industry's output with the market price of its output, or uses some other metric to see how similar the labour content vector is to the monetary output vector. Metrics that have been used are the cosine metric, euclidean distance, mean absolute deviation or correlation. With a cheap PC and publicly published IO tables it takes only a day or two's work to write the code and process the data for an I/O table. The data I present here is obtained from the UK 1998 Input Output table, processed using the programme listed in Section 6.

As an illustration I provide Figures 4.1 and 4.2. The first shows that for the UK there is a strong linear relationship between an industry's direct and indirect labour content, and the monetary value of its total sales. It might be objected that this is just an artifact of industry size (Kliman ((2002))), but it is readily shown that this cannot be an adequate explanation for the relationship.

What one has to do is compare the match one gets with labour content to the match one would get if one chose some input other than labour - for example iron as the basis for value. Using the same procedure one can work back and derive the direct and indirect iron cost of each industry's output. Were the linear relationship shown in Figure 4.1 just the result of industry size, then we should observe a similar relationship for all value bases.

Figure 4.2 shows three other value bases for the UK compared with labour. It is clear that they do not show the simple linear relationship that labour does. Table 1 gives an exhaustive listing of all the other value bases that could be computed using the 1998 IO table, and gives two metrics to compare these with the actual monetary outputs.

Far from being refuted by the evidence it has been confirmed.

Harvey is like a flat earther after Erastosthenes, denying the Earth is round on theological grounds.

### 3 Content

Harvey next complains that Marx nowhere declares his allegiance to the labour theory of value. True enough, since at the time Marx was writing, that was the only theory going. It simply was the theory of value. It was only afterwards that the alternative marginal utility or neo-classical theory of value was established. After Jevons (1866) economists distinguished between the classical or labour theory of value, and the neoclassical or marginalist theory of value. But it is ridiculous to expect Marx to have taken sides in a debate that only started after *Capital* was published (1867). At the time he was writing it was widely accepted that labour was the source of value. Even Jevons the founder of marginalism still accepted that prices were proportional to labour<sup>1</sup>, thinking that his marginal utility theory gave further support to this time honoured assumption.

Harvey promises to explain what Marx's theory of value actually is, but nowhere in his article does he do this. That is because it would be impossible to do this without revealing that the theory of value in Marx is identical in all major predictions to that of Ricardo.

What did Ricardo's theory say?

Did Marx agree with him?

When comparing theorists, especially ones who originally wrote in distinct languages, you should not pay too much attention to the precise vocabulary that they use. What is important is the relations between the concepts they deploy and the relationships that the theorists predict will hold in the real world. When you look at this you see that Marx followed Ricardo's value theory very closely.

There are 4 key components to their value theory on which both authors agree:

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1

"thus we have proved that commodities will exchange in any market in the ratio of the quantities produced by the same quantity of labour." (Jevons (1871) page 187)

1. The exchangeable value of commodities varies with their direct labour content<sup>2,3</sup>.
2. The indirect labour used to make raw materials and equipment also contributes proportionately to the exchange value<sup>4,5</sup>.
3. It is the labour actually expended not the level of pay of the workers that determines value<sup>6,7</sup>.
4. The variation of price with labour will be modified by the formation of an equal rate of profit on stock.<sup>8,9</sup>

The theories are therefore substantially identical in the empirical predictions they make, differing only slightly in terminology. Points 1,2,3 are validated by the empirical data in the studies cited earlier. Point 4 is poorly supported by or refuted by the empirical data Fröhlich ((2013)); Zachariah ((2006)); Cockshott and Cottrell ((1998)) . Marx and

<sup>2</sup>“If the quantity of labour realized in commodities, regulate their exchangeable value, every increase of the quantity of labour must augment the value of that commodity on which it is exercised, as every diminution must lower it.” Ricardo ((1951))Chap. 1 , Sec. 1

<sup>3</sup>“If we consider commodities as values, we consider them exclusively under the single aspect of realized, fixed, or, if you like, crystallized social labour. In this respect they can differ only by representing greater or smaller quantities of labour, as, for example, a greater amount of labour may be worked up in a silken handkerchief than in a brick. But how does one measure quantities of labour? By the time the labour lasts, in measuring the labour by the hour, the day, etc. Of course, to apply this measure, all sorts of labour are reduced to average or simple labour as their unit. We arrive, therefore, at this conclusion. A commodity has a value, because it is a crystallization of social labour. The greatness of its value, or its relative value, depends upon the greater or less amount of that social substance contained in it; that is to say, on the relative mass of labour necessary for its production. The relative values of commodities are, therefore, determined by the respective quantities or amounts of labour, worked up, realized, fixed in them. The correlative quantities of commodities which can be produced in the same time of labour are equal. Or the value of one commodity is to the value of another commodity as the quantity of labour fixed in the one is to the quantity of labour fixed in the other.” (Marx ((1910)), Sec. VI)

<sup>4</sup>“Not only the labour applied immediately to commodities affect their value, but the labour also which is bestowed on the implements, tools, and buildings, with which much labour is assisted”Ricardo ((1951))Chap. 1 Sec. 2

<sup>5</sup>“In calculating the exchangeable value of a commodity we must add to the quantity of labour previously worked up in the raw material of the commodity, and the labour bestowed on the implements, tools, machinery, and buildings, with which such labour is assisted.”(Marx op. cit)

<sup>6</sup>See Ricardo’s criticism of Adam Smith for confusing the labour content of commodities for the labour that a commodity will exchange against.

<sup>7</sup>The whole argument of Marx ((1910)) is devoted to showing that trades unions can raise wages and raise the wage share and that such rises in wages will not simply result in higher prices.

<sup>8</sup>Ricardo ((1951))Chap. 1, Sec. 4

<sup>9</sup>Marx ((1971)) Part II

Ricardo say the sme thing where they are both right and say thre same thing where they are both wrong.

So yes Marx has a labour theory of value as Ricardo had. The greater part of this theory is not refuted by the evidence, it is confirmed by it.

This does not mean that Marx made no contributions. Major innovations in his thought were:

- The point that labour only gets represented as exchange value in societies with private ownership and atomised production. Marx says exchange was absent in traditional Indian communities or the communism of the Incas. The prior economists had assumed that all societies produce commodities.
- The distinction between labour and labour power.
- The introduction of the concept of surplus value as something functionally prior to the division of surplus value between profit, interest and rent.
- A new theory to explain the falling rate of profit.
- A repudiation of Say's law.
- The concept of absolute ground rent.
- The introduction of more modes of production than the ones Adam Smith recognised.
- The idea that the class struggle leads to the dictatorship of the proletariat.
- A more specific identification of productive labour as being productive of surplus value than was made by Smith.

These are all significant innovations that did distinguish him from his predecessors. There is no need to pretend innovation by Marx in value theory, a topic where he just rigorously used Ricardo's concepts.

But what about abstract / concrete labour?

Was this distinction not an innovation on Marx's part?

Well the specific phrase 'concrete labour' was different, but the relevant conceptual distinction between the two was present in Adam Smith's work.

Smith simply uses the term 'labour' unqualified where Marx sometimes says abstract labour<sup>10</sup>. Smith specifically says that when he is talking of labour in this way he is talking of labour in the abstract<sup>11</sup>. When Smith discusses the division of labour, he is discussing the division of the abstract labour into what he terms 'varieties' of labour<sup>12</sup> or 'sorts' of labour<sup>13</sup>. This is the same distinction that Marx is making when, using the

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"A use value, or useful article, therefore, has value only because human labour in the **abstract** has been embodied or materialised in it. How, then, is the magnitude of this value to be measured? Plainly, by the quantity of the value-creating substance, the labour, contained in the article. The quantity of labour, however, is measured by its duration, and labour time in its turn finds its standard in weeks, days, and hours."(Marx ((1954)), page 23 MIA pdf version)

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"The greater part of people, too, understand what is meant by a quantity of a particular commodity, than by a quantity of labour. The one is a plain **palpable** object; the other an **abstract** notion, which though it can be made sufficiently intelligible, is not altogether so natural and obvious." (Smith ((1974)), page 23 Kindle edition)

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"What a variety of labour, too, is necessary in order to produce the tools of the meanest of these workmen! To say nothing of such complicated machines as the ship of the sailor, the mill of the fuller, or even the loom of the weaver, let us consider only what a **variety of labour** is requisite in order to form that very simple machine, the shears with which the shepherd clips the wool. The miner, the builder of the furnace for smelting the ore, the feller of the timber, the burner of the charcoal to be made use of in the smelting-house, the brickmaker, the bricklayer, the workmen who attend the furnace, the millwright, the forger the smith, must all of them join their different arts in order to produce them." .(Smith ((1974)), page 12-13 Kindle edition)

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"The spinner is almost always a distinct person from the weaver; but the ploughman, the harrower, the sower of the seed, and the reaper of the corn, are often the same. The occasions for those different **sorts of labour** returning with the different seasons of the year, it is impossible that one man should be constantly employed in any one of them. This impossibility of making so complete and entire a separation of all the different branches of labour employed in agriculture, is perhaps the reason why the improvement of the productive powers of labour, in this art, does not always keep pace with their improvement in manufactures."Smith ((1974)), page 9-10 Kindle edition)



slightly different talks of abstracting from concrete labours or kinds of labour<sup>14</sup>.

In tabular form we have:

Smith	Marx
variety, kind, palpable	kind, concrete
labour, abstract	abstract-labour

The same conceptual distinction is being made here between the different kinds of activities into which the labour is divided, and labour in the abstract, or the abstract notion of labour.

So both Marx and Smith had the concepts of labour in the abstract although only Marx used the specific term 'abstract-labour'.

We conclude that yes, Marx did have a labour theory of value, and the labour theory of value is the only one with any scientific predictive power.

As to why prices are determined by labour content, a convincing mathematical account was given in Farjoun and Machover ((1983)). Basically the argument is that the a close correlation between prices and labour content is driven by statistical laws which make it the most probable, entropy maximising configuration. But the argument in Farjoun and Machover ((1983)) is dependent on labour being the most universal input into production. Human work enters into every production process, which can not be said of other inputs. This is an outcome of our human ability to perform abstract labour, labour that can be applied to any task.

Smith talks about the labour of animals as well as humans<sup>15</sup>, but Ricardo and Marx

<sup>14</sup>"If then we leave out of consideration the use value of commodities, they have only one common property left, that of being products of labour. But even the product of labour itself has undergone a change in our hands. If we make abstraction from its use value, we make abstraction at the same time from the material elements and shapes that make the product a use value; we see in it no longer a table, a house, yarn, or any other useful thing. Its existence as a material thing is put out of sight. Neither can it any longer be regarded as the product of the labour of the joiner, the mason, the spinner, or of any other definite **kind of productive labour**. Along with the useful qualities of the products themselves, we put out of sight both the useful character of the various **kinds of labour** embodied in them, and the **concrete** forms of that labour; there is nothing left but what is common to them all; all are reduced to one and the same sort of labour, human labour in the abstract." (Marx ((1954)), page 28 MIA pdf version)

<sup>15</sup>"the price of corn, for example, one part pays the rent of the landlord, another pays the wages or maintenance of the labourers and labouring cattle employed in producing it, and the third pays the profit of the farmer. These three parts seem either immediately or ultimately to make up the whole price of corn. A fourth part, it may perhaps be thought is necessary for replacing the stock of the farmer, or for compensating the

do not give beastly labour a role in value. A horse or ox can be set to provide traction, or to carry loads. Human porters can do the same. But not horse can be set to crew a ship, make shoes, or level a course of bricks. Human labour alone has this universal, polymorphic character. We humans are the Real Universal RobotsCapek ((1999)).

## 4 Data

This section presents data obtained from the 1998 UK IO table using the techniques described in Cockshott and Cottrell ((1997)). A slight novelty exists in the metrics used here.

Let  $\cdot$  be the inner product operator, and  $\| \cdot \|$  denote the L2 norm of length of a vector. Let  $M$  be the final output vector of the IO table in money terms. Let  $B_i$  be a vector giving the direct and indirect consumption of the  $i$  th input to produce the final output, with  $i$  ranging over the set of inputs from Agriculture to Labour in the table above. The unit inner product is then given by  $\frac{B_i \cdot M}{\|M\|^2}$  and the normalised inner product by  $\frac{B_i \cdot M}{\|B_i\| \|M\|}$ . In geometric terms the operations involve two different ways of projecting the vector  $B$  onto the hypersphere of unit radius. This is illustrated further in Figure 4.3.

It is interesting to note that the only production input whose ability to predict monetary value comes close to labour in Table 1 is Computer and Related Activities(CRA). The cosine angle between the labour content vector and the final money value vector is 0.98 corresponding to a vector angle of  $11^\circ$  whilst for CRA the cosine is 0.94 corresponding to an angle of  $19^\circ$ .

A first thought is that if, due to automation, computers are replacing humans, then wear and tear of his labouring cattle, and other instruments of husbandry.”(Smith ((1974)), Page 33, Kindle edition)

“No equal capital puts into motion a greater quantity of productive labour than that of the farmer. Not only his labouring servants, but his labouring cattle, are productive labourers. In agriculture, too, Nature labours along with man; and though her labour costs no expense, its produce has its value, as well as that of the most expensive workmen. The most important operations of agriculture seem intended, not so much to increase, though they do that too, as to direct the fertility of Nature towards the production of the plants most profitable to man.”(Smith ((1974)), Page 196, Kindle edition)

**Table 1:** Comparison of different value bases for the UK in 1998.. Note that none of the alternative value bases approaches labour in predictive power.

Value Base	Unit Inner Product	Normalised Inner Product
<i>i</i>		
AGRICULTURE, HUNTING, FORESTRY AND FISHING	2%	37%
MINING AND QUARRYING	2%	47%
FOOD PRODUCTS, BEVERAGES AND TOBACCO	3%	61%
TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR	1%	52%
WOOD AND PRODUCTS OF WOOD AND CORK	1%	50%
PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING	3%	60%
COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL	3%	83%
CHEMICALS EXCLUDING PHARMACEUTICALS	2%	60%
PHARMACEUTICALS	1%	39%
RUBBER AND PLASTICS PRODUCTS	2%	80%
OTHER NON-METALLIC MINERAL PRODUCTS	2%	51%
IRON & STEEL	1%	40%
NON-FERROUS METALS	1%	46%
FABRICATED METAL PRODUCTS, except machinery and equipment	2%	59%
MACHINERY AND EQUIPMENT, N.E.C.	1%	61%
OFFICE, ACCOUNTING AND COMPUTING MACHINERY	1%	24%
ELECTRICAL MACHINERY AND APPARATUS, NEC	1%	51%
RADIO, TELEVISION AND COMMUNICATION EQUIPMENT	1%	44%
MEDICAL, PRECISION AND OPTICAL INSTRUMENTS	1%	56%
MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS	2%	43%
BUILDING AND REPAIRING OF SHIPS AND BOATS	1%	45%
AIRCRAFT AND SPACECRAFT	1%	14%
RAILROAD EQUIPMENT AND TRANSPORT EQUIPMENT N.E.C.	0%	61%
MANUFACTURING NEC; RECYCLING	1%	73%
ELECTRICITY, GAS AND WATER SUPPLY	2%	39%
CONSTRUCTION	4%	42%
WHOLESALE AND RETAIL TRADE; REPAIRS	4%	90%
HOTELS AND RESTAURANTS	1%	89%
TRANSPORT AND STORAGE	8%	79%
POST AND TELECOMMUNICATIONS	3%	84%
RENTING OF MACHINERY AND EQUIPMENT	2%	79%
COMPUTER AND RELATED ACTIVITIES	3%	94%
RESEARCH AND DEVELOPMENT	1%	84%
OTHER BUSINESS ACTIVITIES	11%	90%
EDUCATION	1%	80%
HEALTH AND SOCIAL WORK	5%	38%
OTHER COMMUNITY, SOCIAL AND PERSONAL SERVICES	2%	61%
LABOUR	51%	98%
MARKET VALUE	100%	100%

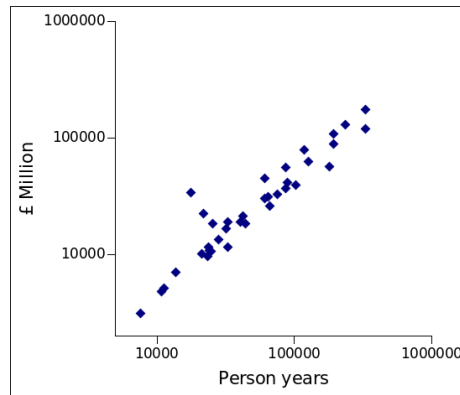


Figure 4.1: There is a clear linear relationship between the monetary value of UK industry outputs and the labour content of these outputs. Each dot represents an industry, the X axis is scaled in the person years that directly or indirectly went into the producing the output of that industry. The Y axis is the value in money terms of the industry output. Results are computed from the UK 1998 Input Output table.

computer inputs should end up correlating well with monetary outputs. On top of that the computers we use today are Universal Computers derived from the pioneering work of Turing ((1937, 2004, 1950)). This means that they can be set to solve any mathematical problem, the same model of processing chip can be used for thousands of different calculation, control and monitoring purposes. They are thus the only artificial machine that approaches - albeit partially the universality of human labour. It thus seems appropriate that the only empirical contender as source of value is computer services. But we should not be over keen to herald the replacement of humanity by computers. This is just one nation for one year. The results for labour have been replicated over many nations and years, and there is contradictory information in the 1998 UK table: the vector for Office Accounting and Computing Machinery has a cosine of only 0.24 corresponding to an angle of  $76^\circ$  to the price vector. So computer hardware, the actual universal computers, have only a minor influence on monetary value. It is possible that the line in the 1998 table called Computer and Related Activities actually refers to the employment of software another product of mainly human labour. It is also worth noting that the relative effect of CRA is likely to be low since

the Unit Inner Product with the output vector is only 3% as against 51% for labour. This indicates that whilst the direction of the CRA vector is close to the direction of the final output, it can have only a vestigial effect on final sales valuation. Instead the alignment is likely to be a second order effect of the fact that such a large part of the workforce now work using computers, so expenditure on computer software licenses will track the number of workers using, for example, workstations running Windows.

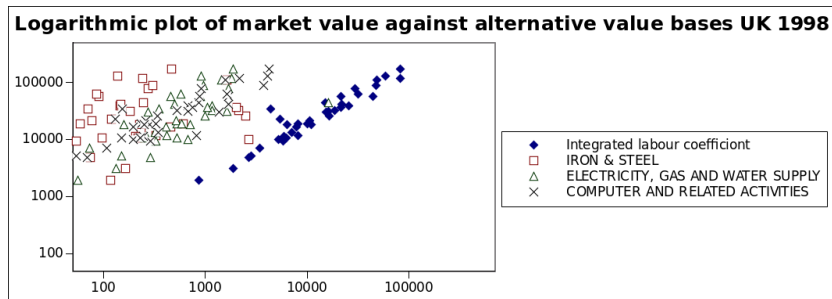


Figure 4.2: Other possible value bases perform much less well than labour. Here three other widely used industrial inputs are compared with labour to see how well they predict selling prices. The X axis in this case is the integrated cost in £1 million spent directly or indirectly by each industry on the product in question. It can be seen that the other inputs fail to show the same linear relationship to output value.

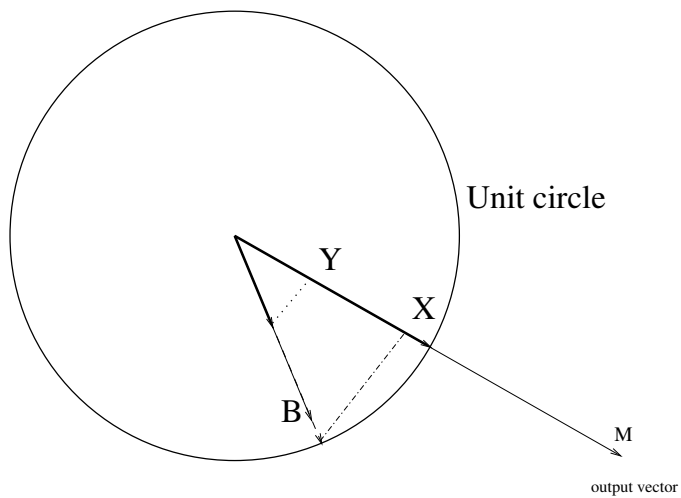


Figure 4.3: Projections of vectors onto the unit circle or unit hypersphere. Recall that the inner product projects one vector onto another.  $M$  is a vector of output measured in money prices.  $B$  the corresponding vector in some other base,  $Y$  is what I have called the Unit Inner Product,  $X$  is what I have called the Normalised Inner Product. The Normalised Inner Product is equal to the cosine of the angle between the two vectors and is a pure measure of the direction that the vectors point. The Unit Inner Product scales the vector  $B$  by  $1/\|M\|$  before projecting it onto the unit vector pointing in the direction  $M$ . As such it takes into account both the angle between and the relative lengths of the two vectors  $M$  and  $B$ .

## 5 Conclusion

The meaning of scientific theories come down to the specific causal relations that they say exist in the world. Harvey claims that Marx rejected the Ricardian theory of value, now called the labour theory of value, and that his alleged rejection was justified because that Ricardian theory is completely discredited.

We first showed, by textual references, that the causal relations between entities and observables in the value theories of Marx and Ricardo are the same. Thus, contrary to Harvey's claims, Marx upheld substantially the same theory as Ricardo.

Secondly we cited a corpus of econometric literature which show that the labour theory is in fact true.

Finally we give a concrete illustration of these econometric methods by demonstrating that the monetary value of outputs of 36 industrial sectors in the UK is well predicted by the direct and indirect labour used by these sectors. Further, we showed that no other input to production comes close to the influence that labour exerts on the value of output.

## 6 Appendix: The programme used

```
program labourValuation ;  
uses csvfilereader ;
```

### 6.1 Purpose

The Vector PascalCockshott and Michaelson ((2006)); Cockshott ((2002)) programme below extracts labour values from input output tables.

This programme can potentially be used in studies of the comparative efficiency of



labour values, prices of production or Kantorovich valuations in economic planning.

## 6.2 Comand line interface

**Usage :** labourValuation *iotable.csv*

The files should be laid out with a first line and first column made up of text fields labeling the rows and columns. All other cells should be numeric. Discussion of matrix sizes in what follows refers exclusively to the rectangular subarray of numeric values.

### 6.2.1 iotable

The first file, the iotable one, should contain an N by M io table in standard column form, with a column corresponding to an industry so that cell at row i col j contains the amount of output from the ith industry used by the jth industry.

The last line must contain the outputs of each industry.

## 6.3 Code

The programme reads in the input output table using a library for accessing .csv files. It then uses a Jacobi solver to extract labour values.

```
type pmat=^matrix;
channel= record
    p:pcsv;
    r,c:pheadervec;
    m:^matrix;
end;
procedure rf(var ch:channel;param:integer );
(*! Read in one of the file parameters and extract the data from it *)
begin
    with ch do
```

```

begin
  p:=parsecsvfile(paramstr(param));
  if p=nil then
    begin
      writeln('error opening or parsing file ',paramstr(param));
      halt(2);
    end
  else ;
    r:=getrowheaders(p);
    c:=getcolheaders(p);
    m:=getdatamatrix(p);
  end;
end;
var iot:channel;
  L,V,0,01:^vector;
  square:^matrix;
  i,j:integer;
begin {main programme}
  rf(iot,1);
  new(L,iot.m^.cols);new(V,iot.m^.cols);
  new(0,iot.m^.cols);new(01,iot.m^.cols);
  {initialise all labour values to 1}
  L^:=1;
  { extract the variable capital vector from the IO table}
  V^:=iot.m^ [iot.m^.rows-1];
  {Extract the final output price vector from the IO table}
  0^:=iot.m^ [iot.m^.rows];

```

```

{Create a transposed version of the IO table central matrix}
new(square,iot.m^.cols,iot.m^.cols);square^ := trans (iot.m^);
{Jacobi solution of labour values, 20 iterations is
  enough to give a very accurate estimate }
for i:= 1 to 20 do
begin
  O1^:= V^+ \+ (L^ * square ^);
  L^:= O1^/O^;
end;
writeln(O1^:12:1,O^:12:1);
end.

```

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